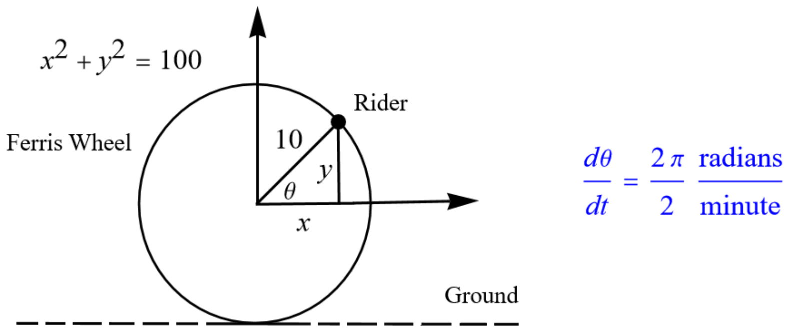
Exercise 46

A Ferris wheel with a radius of 10 m is rotating at a rate of one revolution every 2 minutes. How fast is a rider rising when his seat is 16 m above ground level?

Solution

Draw a schematic of the rider's path at a certain time.



The aim is to find dy/dt when $y = 6$. Use a trigonometric function to relate the angle θ with convenient sides of the triangle.

$$\sin\theta = \frac{y}{10}$$

Solve for y.

$$y = 10\sin\theta$$

Take the derivative of both sides with respect to time by using the chain rule.

$$\frac{d}{dt}(y) = \frac{d}{dt}(10\sin\theta)$$

$$\frac{dy}{dt} = (10\cos\theta)\cdot\frac{d\theta}{dt}$$

$$= 10\left(\frac{x}{10}\right)\cdot\left(\frac{2\pi}{2}\right)$$

$$= x\cdot\pi$$

$$= \left(\pm\sqrt{100 - y^2}\right)\cdot\pi$$

Since we want to know the rate at which the rider is rising (as opposed to falling), we choose the plus sign.

$$\frac{dy}{dt} = \pi\sqrt{100 - y^2}$$

Therefore, when the rider is 16 feet above the ground, the rate that he's rising with respect to time is

$$\left.\frac{dy}{dt}\right|_{y=6} = \pi\sqrt{100 - (6)^2} = 8\pi\ \frac{\text{m}}{\text{min}} \approx 25.1327\ \frac{\text{m}}{\text{min}}.$$